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THE BEGINNING OF MATHEMATICS¹

By W J MCGEE

I

Chemistry grew out of alchemy as natural experience waxed and primeval mysticism waned in reciprocal measure; and in earlier time astronomy grew out of astrology in similar fashion. The growth of chemistry is fairly written and that of astronomy meagerly recorded in early literature; and in the history of both sciences the records are corroborated and the sequence established by vestigial characteristics, which are no less useful in defining mental development than are vestigial organs and functions in outlining vital evolution.

The beginning of chemistry marked the third step in the development of science; the beginning of astronomy marked the second step; the first step, taken amid the mists of unwritten antiquity, was marked by the beginning of mathematics. In the absence of records, the rise of mathematics may be traced partly (like that of the next younger sciences) by vestigial characteristics; and these characteristics indicate that, just as scientific chemistry came out of mystical alchemy, and as scientific astronomy sprang from mystical astrology, so scientific mathematics grew out of a mystical system which long dominated the minds of men and slowly waned under the light of natural experience concentrated among the Arabs of past millenniums. In Arabia this mystical system preceded the essentially natural (though happily conventional) system of algorism, which opened the way to numerical treatment of quantities, and thus gave a foundation

¹ Read before the American Association for the Advancement of Science (Section H) at Columbus, August 22, 1899.

for science; it even antedated conventional algebra, in which symbols are used to represent natural values, and seems to have dropped into the background of thought with that long-abandoned side of algebra, *almacabala* (or *almachabel*)—a jumble of occult or semi-occult redintegration which appeals strongly to the half-developed mind. So the stepping-stones to science may be enumerated as *almacabala*, astrology, alchemy, leading respectively to mathematics and astronomy and chemistry, the oldest branches of definite knowledge.

While the transition from *almacabala* to mathematics is indicated by vestigial characteristics among the peoples influenced by Arabic culture (the Aryans and their associates, who make up the intellectual world), the sequence is established by parallel phenomena displayed by other lines of culture. The import of these parallel phenomena becomes clear in the light of cardinal principles, pertaining both to science in general and to anthropology in particular: (1) In all science it is necessarily postulated that knowledge grows by successive increments, through experience and its assimilation, through observation and comparison (or generalization), through discovery and invention—i. e., through natural processes; (2) in all branches of definite knowledge, but especially in anthropology, it is implicitly if not explicitly postulated that knowledge is diffused and its acquisition stimulated through association and interchange among individuals and peoples; (3) in anthropology, as in other sciences, it is necessary to recognize a body or volume of knowledge proper to each people, made up of the combined possessions of all the individuals, increasing with successive experiences, decreasing only through neglect or disuse, and in greater part perpetuated by record and tradition if not by direct heritage; (4) in anthropology, as measurably in other sciences, it is desirable to assume that (*a*) mental capacity and (*b*) the sum of knowledge, either in the individual or in the group, are in the long run practically equivalent; and (5) in anthropology it is

convenient to assume that the course of mental development is approximately uniform (or about as nearly similar as are environmental conditions) in each separate or independent group of men—this assumption, which is rapidly crystallizing in the minds of anthropologists, being but a corollary of the primary postulate on which all science rests, namely, that knowledge grows by natural means.

The mere recognition of these principles renders it clear that the particular growth-stage of any intellectual stock (or people) may be defined with approximate accuracy by gauging the mentality of other peoples developed to corresponding stage, just as the history of the aged sequoia grove of prehistoric birth may be read in terms of younger groves on neighboring ranges; for the towering forests of the big-tree species and the upshooting forests of human ideas may well be likened in individual and collective growth, save that the vegetal species is decadent and shrinking into separate groves in scattered holts, while the mental growth is luxuriant and spreading exuberantly from province to province throughout the lands of the earth. The interpretation in terms of growth-stages is established by conformity with natural law; did the forest receive extra-natural impulse at any stage, or did knowledge arise otherwise than through interaction with nature, the interpretation would fail; but in the absence of evidence against the uniformity of nature, the equivalence of corresponding stages must be recognized alike for the figurative forests of ideas and the material forests of wood and leafage.

The acceptance of these cardinal principles affords a means of tracing the unrecorded history of Aryan culture, and of interpreting the meager records of Arabia's mathematical pioneering in terms of the culture of other peoples still below, or just rising upon, the plane marked by the birth of writing. Especially useful for comparison are various practically independent Amerindian peoples, some low in prescriptorial culture, others trembling on the verge of definite graphic art, and still others within

that phase of scriptorial culture marked by conventional calendric and numerical systems; hardly less useful are several African peoples representing various stages of development; equally significant, too, are the Australasian tribes of culture so low that numerical knowledge is inchoate only; while useful suggestions as to the origin of numerical concepts may be obtained from various subhuman animals. True, the lines of growth maturing in mathematical systems must vary with environmental conditions, and doubtless with hereditary traits persistently reflecting both ancestral and proto-environmental factors; yet, if knowledge be not an extra-natural product rather than a reflex of nature (as brilliantly conceived by Bacon) the lines must be so far conformable as to render the comparisons trustworthy and sufficiently accurate for practical purposes—just as the retracing of the history of an isolated grove by comparison with the growth-lines of other groves must be inexact in detail, though trustworthy in general and sufficiently accurate to meet practical needs.

II

In tracing lines of activity maturing in civilization and enlightenment, it is needful to note certain habits of mind characteristic of all primitive men; and for present purposes (as for practically all others), it suffices to define primitive peoples as those who have not yet acquired and assimilated the art of writing—i. e., as those who remain in prescriptorial culture:

I. Primitive men are mystics. Believers in extra-natural potencies, skillless observers, and inconstant reasoners, their faith counterfeits realities, and clothes its own figments with all manner of attributes, both appropriate and incongruous. In their simple (and presumptively primeval) aspect, the fear-born figments are grotesque shadows or fantastic duplicates of actual things, moved by caprice or malice like unto that of human kind; grown more complex, the figments are incarnated chiefly in

self-moving things and invested with intensified autonomy ; and in the higher stages of primitive culture, they are idealized into mystical potencies actuating the phenomena of the universe in accordance with impulses and motives reflecting those of the primitive mind. In all its aspects, the faith is profound ; it is an ever-present possession, passing even into complete obsession, whereby thought and action are habitually and wholly controlled.

In every phase of primitive culture, the mystical potencies are deemed chief factors of failure or success in the ceaseless strife for existence ; they are invoked by fasting, propitiated by sacrifice, celebrated by feasting, and generally expiated by individual and collective ceremony and by the marvelously persistent tradition of prescriptorial culture ; and since the successful men and tribes give more thought to joyous glorification and less to anxious propitiation than their unsuccessful contemporaries, the beneficent potencies tend to survive and the maleficent mysteries tend to die out of the darksome—yet ever brightening—faith of primitive men.

In every stage of primitive faith the controlling mysteries are associated with symbolic objects and actions, and both mysteries and symbols are zealously enshrouded in deeper mystery. So fetishism and shamanism grow apace, and not only ceremonial objects but places and persons and forms of utterance become secret or sacred ; esoteric observances, impressive insignia, and imposing formalities are established ; and systems of rank or caste grow up as tangible expressions of the intangible subjective structures. Cumulatively strengthened by reaction of symbol on mystery and of mystery again on symbol, the pervading mysticism is intensified above all other motives in the primitive mind ; and the artistic concepts, the industrial devices, the social relations, and the subjects and forms of speech pass under the control of the unreal potencies which shadow all primitive thinkers.

Throughout primitive culture, invocation habitually carries an obverse of incantation, and the normal course of fiducial development is at-

tended by persistent sortilege or thaumaturgy ; and in the higher strata necromancy and soothsaying, spells and enchantments, conjury and exorcism, oracles and ordeals, and divination by lot or chance, become characteristic. In the higher strata, too, expressions supplement or supplant the objective symbols of lower plane, and the jargon of jugglers and the farrago of fakirs take the place of fetishes and idols ; and it is particularly significant that words and verbal formulas come to be regarded as superpotent expressions of mystical power. Some savage tribes regard their language as sacred, some have hieratic languages, and among all known tribes personal names are considered magical or tabu in one way or another ; while just within the lower strata of scriptorial culture (as illustrated by the Arabs and Hindoos and other Eurasians of a few centuries ago, and attested by literary and linguistic and objective vestiges), shibboleths and numerical formulas become rife, and the inscribed talisman and abracadabra and mystical number, and eventually the magic square, form favorite symbols of occult power.

With the growth of writing and the decadence of tradition, mysticism began to lose control of the human mind ; but innumerable vestiges along the line of Aryan culture, as well as the examples afforded by other lines, demonstrate the potency of this primeval factor and the tenacity of its hold on the human mind.

2. Primitive men are egoists. Knowing little of the external world, tribesmen erect themselves or their groups into centers about which all other things revolve according to the caprice of their all-potent mysteries ; they act and think in terms of a dominant personality, always reducible to the Ego, and an Ego drawn so large as to stand for person, place, time, mode of action, and perhaps for *raison d'être*—it is Self, Here, Now, Thus, and Because. Science shows that the solar system hurtles through space, presumably about an unknown center ; it showed before that the sun is the center of our system ; but the heliocentric system was expanded out of an antecedent geocentric

system, itself the offspring of a democentric system, which sprang from an earlier ethnocentric system born of the primeval egocentric cosmos of inchoate thinking.

In higher culture the recognized cosmos lies in the background of thought, at least among the great majority; but in primitive culture the egocentric and ethnocentric views are ever-present and always-dominant factors of both mentation and action. Their prominence is exemplified by kinship organization, the universal basis of primitive society; and this organization cannot better be illustrated than by analogy with the planetary assemblage: Each individual rotates independently, may be attended by satellites, and revolves first about the head of the family and later about the patriarch of the group, yet each exerts a definite attractional influence (albeit proportional to individuality rather than mass) on all his associates; and the relative social positions are expressed and kept in mind by habitual conduct and form of speech. The primitive man cannot speak to or of a companion without reference to the currently-accepted view of his circumscribed cosmos—he cannot say “brother,” but must say my “elder brother,” or use some other term implying relative position of several individuals to himself, and among each other as reckoned through himself.

Naturally the ever-present view of a self-centered cosmos finds expression throughout language and faith, and in arts and industries, as well as in social organization: Primitive language is essentially associative, abounding in numbers and genders, persons and cases, tenses and moods, in a peculiarly complex structure reflecting the egocentric habit of thought, so that primitive grammar is chaotically elaborate; and in primitive belief the individual long retains his personal tutelary or fetish, endowing it with characters revealing his own subjectivity, and only slowly rises first to the recognition of family fetishes and clan totems, and much later to that recognition of alien tutelaries which ends in pantheism. Concordantly, primitive art is conspicuously egoistic, beginning typically with the

totemic face-paint or tattoo-mark of the matron, or with the autobiographic calendar of the shaman; while the industrial devices of early culture are held to absorb and retain a part of the personality of, and indeed to become subjective appendages to, their makers and users.

So self-centered thinking is crystallized by custom, and the thought and custom interact with cumulative effect in dominating the primitive mind well into the upper strata of prescriptorial life; and numberless vestiges of egocentric cosmology cling even unto the higher phases of Aryan culture.

It cannot be too strongly emphasized that primitive thought is unlike the finer product of contemporary intellectuality: The most conspicuous differences are connected with the pervading mysticism and prevailing egoism of primitive thinkers, which are magnified in their influence by the fewness of concurrent intellectual motives; so that prescriptorial culture may justly be regarded as the outgrowth and outshowing of that mysticism-egoism which began to decline with the birth of writing yet still retains some hold on the minds of men.

III

Simple counting is an accomplishment common to men and many lower animals. The special appreciation of numbers sometimes displayed by horses, dogs, and pigs may be due to human association, while the geometric sense of the bee may be considered mechanical merely; yet the well-known ability of the crow to count (or at least to discriminate units) up to six or seven, the similar faculty of the fox, and the habits of wasps in providing fixed numbers of spiders for their unborn progeny, as well as various other examples, demonstrate a native capacity for numerical concepts on the part of birds and mammals and insects. Apparently similar is the numerical capacity of various lowly tribes of different continents: Numerous Australian tribes are described as counting laboriously up to two, three, four, or

six, sometimes doubling two to make four or three to make six, and in other ways revealing a quasi binary system, though both Curr and Conant opine that "no Australian in his wild state could ever count intelligently to 7"¹; certain Brazilian tribes are also described as counting only to two, three, or four, usually with an additional term for many; while the Tasmanians counted commonly to two and sometimes to four, and were able to reach five by the addition of one to the limit number.²

The analogy between the counting of the tribesmen and that of the animals is not so close as the bare records suggest, since the descriptions of the tribal reckoning relate to systems of vocal numeration rather than to actual ability in discrimination and enumeration; moreover, most of the tribesmen reveal the germ of notation in the use of sticks, notches, knotted cords, and the like to make tangible the numerical values, something which lower animals never do so far as known. Actually the savages, even those of lowliest culture, habitually count to or above three, as shown by the plurality of plurals and other features of their speech; and the meagerness of their numeration no more negates numerical capacity than does the absence of such systems among counting crows and foxes and wasps. Nevertheless, the comparison is instructive; in the first place, it indicates roughly corresponding ability to count on the part of higher animals and lower men; it also defines the origin of vocal numeration at the bottom of the scale of human development; and it is especially significant in demonstrating that neither the animals nor the men either (1) cognize quinary and decimal systems, or (2) use their own external organs (toes, fingers, etc.) as mechanical adjuncts to nascent notation—unless the binary numeration of certain Australian tribes is really bimanual, as W. E. Roth implies.³ Many

¹ *The Number Concept*, by L. L. Conant, 1896, p. 27; *The Australian Race*, by E. M. Curr, 1886, vol. I, p. 32.

² *The Aborigines of Tasmania*, by H. Ling Roth, 1890, p. 147.

³ *Ethnological Studies among the North-West-Central Queensland Aborigines*, 1897, p. 2.

primitive peoples count by fingers and hands, sometimes with the addition of toes and feet, and thereby fix quinary, decimal, and vigesimal systems; but the burden of the evidence derived from animal counting and the numeration of lower savagery seems to demonstrate that these systems are far from primeval.

Simple number systems of mystical or symbolic character abound among the better-studied tribes of middle-primitive culture, including the aborigines of North America. The most widespread of the mystical numbers is four; it finds expression in Cults of the Quarters in North America, South America, Asia, and Africa, and is suggested by certain customs in Australia¹; it is crystallized in the swastika or fylfot and other cruciform symbols on every continent; and it is established and perpetuated by associations with colors, social organization, and various customs among numerous tribes. In much of primitive culture the hold of the quatern concept is so strong as to dominate thought and action—so strong as to seem wholly inexplicable save through the interwoven mysticism and egoism of the lowly mind. The devotee of the Cult of the Quarters is unable to think or speak without habitual reference to the cardinal points; and when the quadrature is extended from space to time, as among the Papago Indians, the concept is so strong as to enthrall thought and enchain action beyond all realistic motives. To most of the devotees of the quatern concept—forming probably the majority of the middle-primitive tribes of the earth—the mystical number four is sacred, perfect, all-potent, of a perfection and potency far exceeding that of six to the Pythagoreans and the hexagram to Paracelsus; they are unconscious or only vaguely conscious of any other numerical concept; and many investigators fail to discover the obverse of the quartered shield and trace the mystical figure to the subconscious Self which it invariably reflects. Yet careful inquiry shows that the cardinal points are never conceived apart from the Ego in the center; that the subjectively prepotent

¹ *The Australian Race*, op. cit., vol. I, pp. 339, 340.

part of the swastika is the intersection, or common origin, of the arms; that the four colors of brightening sunrise, and boreal cold, and blushing sunset, and zephyr-borne warmth must have a complementary all-color in the middle; that the four winds are balanced against some mythic storm-king (able to paralyze their powers in response to suitable sacrament) in or near the Middle of the world; that the sky falls off in all directions from above the central home of the Real Men; that the four termini of Papago time relate to the end of the period conceived always with respect to the beginning; that the four worlds of widespread Amerindian mythology comprise two above and two below the fate-shadowed one on which the shamans have their half-apperceived existence; that the four phratries or societies are arranged about the real tribal center; and that in all cases the exoterically mystical number carries an esoteric complement in the form of a simple unity reflecting the egoistic personality or subjectivity of the thinker. It is easier to represent the quatern concept graphically than verbally—indeed it has been represented graphically by unnumbered thousands of primitive thinkers in the cruciform symbols dotting the whole of human history and diffused in every human province, or in the form of the equally widespread but less conspicuous quincunx.

The exoterically quatern and esoterically quincuncial concept appears to mark a fairly definite phase of human development; a somewhat higher stage is marked by the use of six as a mystical or sacred number. In this stage the mythology remains a Cult of the Quarters, though the cardinal points are augmented by the addition of zenith and nadir, while a third upper-world and a third under-world are usually added. The ramifications of the concept are still more extended than those of the quatern idea, and lead to even more patent incongruities—particularly when the attempt is made to graphically depict the essentially tri-dimensional concept on a plane. Now the senary concept, like its simpler analogue, is always incomplete in itself: The six cardinal

points must be reckoned from a common center, the three under-worlds and the three upper-worlds are reckoned from the Middle world of actuality, and the six colors (e. g., of corn, as among the Zuni according to Cushing and others) are habitually supplemented by a central all-color; so that, in this case as in the last, the exoterically perfect number is esoterically perfected through the unity of subjective personality.¹ The six-cult is much less extensively distributed through history and throughout the world than the four-cult, but may be traced in different continents; and it is peculiarly meaningful in establishing that marvelous prepotency of the number-cult which, among many tribes, carried the nascent numeral system past the point at which nature strove, through obvious organic structure and algorismic order, to implant the quinary system—indeed if further evidence than that of bestial and savage counting were required to show that finger-numeration and the quinary system were not primeval, it would be afforded by the development of the senary-septenary system in so many lands.

The quaternary and senary cults illumine the binary systems prevailing among tribes still lower in the scale of intellectual development. Especially helpful is the light on the Australian aborigines, who are found thereby to exemplify what might be called a Cult of the Halves; for their binary concept of things is expressed not only by their numeration, but even more clearly by their social and fiducial systems, which, in turn, shape their everyday conduct and speech.

“The fundamental feature in the organization of the Central Australian, as in that of other Australian tribes, is the division of the tribe into two exogamous inter-marrying groups,” say Spen-

¹ The perfecting of the mystical numbers four and six by the addition of unity has been recognized by many investigators, notably Powell (*On Regimentation*; 15th Annual Report of the Bureau of Ethnology, 1893-'94, 1897, p. cxvii and elsewhere), Morris (*Relation of the Pentagonal Dodecahedron . . . to Shamanism*; Proceedings of the American Philosophical Society, vol. XXXVI, 1897, pp. 179-183), and Cushing (*ibid.*, p. 185 and elsewhere).

cer and Gillin¹; and all other students of native Australian society have either been overwhelmed by an apparently irresolvable nebula of overlapping classes and sub-classes and super-classes, or have been led to a related conclusion—indeed the gordian knot of entangled relationships constituting Australian society is easily cut by the student who places himself in the position of an individual Blackfellow, and projects from Self dichotomous class-lines occasionally uniting and bifurcating in other individuals, after the manner of the dichotomous lines of Aristotelian classification and the Tree of Porphyry. The social classes, and the conduct involved in their maintenance, are fixed by a bifurcate series of ordinances, ostensibly descended from the mystical olden time, and put in the form of tabus and equally mystical mandates by the shamans. In like manner the obscure pantheon of the Australians seems to be arranged in nearly symmetric pairs; and even the individual shade (or mystical double of the person) is conceived as bipartite—e. g., among the Arunta, who designate the ghostly attendants Iruntarinia and Arumbaringa, respectively.²

Although typically developed among the Australian aborigines, the binary philosophy is by no means confined to the Austral continent and primeval culture; it existed among the Tasmanians, reappears in Africa, persists in China and Mongolia, and may clearly be traced in America, e. g., in the “Sides” forming the primary basis of society in the Seneka and other Amerind tribes; while no fiducial system is wholly free from the persistent dualism springing from binary interpretations of nature. Yet the mystical Two is no more complete in itself than the mystical Four and Six of higher culture; the primary classes or sides are perfected in the tribe both in Australia and in America, the Iruntarinia and Arumbaringa are conjoined in and non-existent apart from the personality they are held to shadow, and the mandates and prohibitions of Australian (and indeed of most other) laws are perfected

¹ *The Native Tribes of Central Australia*, by Baldwin Spencer and F. J. Gillin, 1899, p. 55.

² *Op. cit.*, p. 513.

in permissive or normal conduct ; so that the exoterically binary system of thought is esoterically, or in subconscious fact, ternary.¹

The dichotomous fiducial and social structure clarifies the Australian numeral system. The abundant numerations recorded by Curr and others strongly suggest the simple binary system traced by Conant. A common form is *goona*, *barkoola*, *barkoola-goona*, *barkoola-barkoola* (1, 2, 2-1, 2-2) sometimes followed by "many" or "plenty" and more rarely by *barkoola-barkoola-goona* (2-2-1), though usually the table does not go beyond the fourth term, which may itself be replaced by "many." Now, examination of the numerous records shows (1) that none of the terms correspond with fingers ; (2) that a very few of the terms correspond with the word for hand, such terms being three, four, one, and two in (approximate) order of frequency ; (3) that a somewhat larger number of terms, chiefly three, one, and two, correspond with the words for man ; (4) that a considerable number of threes and ones, with a few fours and twos, suggest affinities with obscure roots used chiefly in terms for man, tribe, wild dog, I, yes, etc. ; and (5) that there is a strong tendency to limit the formal numeration to three. It is particularly noticeable, too, that certain persistent number terms are used sometimes for two and sometimes for three among numerous slightly related tribes—i. e., the term is more definitely crystallized than the concept, which oscillates indiscriminately between two and three, betraying confusion impossible to arithmetic thought. Similarly the Tasmanian numerations are binary, and without reference to finger or hand, though five sometimes appears to connote man. These features clearly indicate that the Australasians do not count on their fingers, and are without realistic notion as to the number of fingers—indeed, the Pitta-Pitta of Queensland are able to count their fingers and toes only by the aid of marks in the sand,² while the abundant Australian pictographs reveal habitual

¹ Lumholtz mentions a definite "idea of the Trinity" among the southeastern Australians (*Among Cannibals*, 1889, p. 129).

² *Ethnological Studies*, by Walter E. Roth, *op. cit.*, p. 26.

uncertainty as to the number of fingers in the human hand (save where the picture is developed from a direct impression).¹

In the dearth of knowledge concerning the original or collateral meanings of the Australian number-terms, it is difficult to formulate the concept or give it graphic expression; but a suggestion of great inherent interest is found in the Shahaptian numeration, in which, according to Hewitt, the first two integer-terms are denotive or arbitrary merely, while the term for three means Middle or Middle ONE—not middle finger or middle of the hand, but apparently a general (or semi-abstract) Middle like that of the Zuni ritual; and the suggestion is enforced by corresponding expressions in Serian, Iroquoian, and some other Amerindian tongues. In the light of these analogies, the Australian thought-mode, with its numerical and social and fiducial expressions, assumes definite and harmonious shape in a binary-ternary system in which things are arranged in pairs and related subconsciously to the Ego as an interpretative nucleus.




The three number-systems pertaining to prescriptorial culture are essentially distinct from Aryan arithmetic, both in motive

¹ Suggestively analogous in form and meaning are certain South American number systems, e. g., that of the Toba, whose ordinary numeration ends with six (the term meaning also "many" or "plenty"), though Bárcena has traced it to ten. The terms are somewhat variable, and of such form as to imply actual or vestigial connotive character; as recorded by Quevedo (*Arte de la Lengua Toba*, por el Padre Alonso Bárcena . . . con Vocabularios . . . por Samuel A. Lafone Quevedo, Biblioteca Lingüística del Museo de la Plata, tomo II, 1898, p. 41) they are *nathedac*, *cacayni* or *nivoca*, *cacaynilia*, *nalotapegat*, *nivoca cacainilia* ($2 + 3$), *cacayni cacaynilia* (2×3), *nathedac cacayni cacaynilia* ($1 + 2 \times 3$), *nivoca nalotapegat* (2×4), *nivoca nalotapegat nathedac* ($2 \times 4 + 1$), *cacayni nivoca nalotapegat* ($2 \times 4 + 2$). Now, it is noteworthy (1) that none of the terms connotes finger, hand, or man; (2) that there are alternative terms for two in both simple and composite uses; (3) that two is the most prominent factor in the composite part of the series; (4) that one of the terms for two and the term for three are closely similar, and distinguished only by inflection; (5) that the term for four apparently connotes equality (*nalotath* = equal) and declaration (*na-pegá* = they say; *sena-pegá* = I say, etc.); and (6) that the system is definitively not quinary or decimal. There are suggestions, both in the combinations and connotations of the terms, of two threes of ill-defined numerical character, corresponding respectively to the numerical 2 and 3; and that four is an essentially mechanical square. There are also many indications that the system is inchoate so far as the strictly numerical aspect is concerned.

and mechanism. Primarily they are devices for divination, or for binding the real world to the supernal, and it is only later or in ancillary way that they are prostituted to practical uses; yet by reason of the extra-natural potency imputed to them they dominate thought and action in the culture stages to which they belong, and profoundly affect the course of intellectual development. The three systems correspond in that each rests on an exoteric base in the form of an even number, and in that each is really governed by a half-apperceived unity, itself the reflection of the Ego; they differ in the value of the exoteric base, itself a measure of the intellectual capacity normal to the culture stage to which it pertains. The two higher systems have graphic equivalents which intensify and measurably shape their mystical potency (for the mechanical conditions attending graphic representation always interact with primary concepts in primitive thought); but the lowest and presumptively primeval system is without known graphic symbol.






Naturally, number-systems resting on inconstant and largely subjective bases are not susceptible of treatment in accordance with rational arithmetic; but the two higher systems (and probably the lowest also) are susceptible of combination in accordance with a *law of augmentation*, which is neither addition nor multiplication, but which tends to generate both; and this law of augmentation is significant (1) as indicating the evolution of number systems both mystical and rational, and (2) as a source of those vestigial features of *almacabala* still persisting in Aryan culture.

The augmentation of the widely diffused quaternary-quinary system is made clear by aid of its mechanical symbolism, which combined with the egoistic concept to shape the system. The commonest (and nearly world-wide) symbol is the cruciform figure † , or the quincunx, $\cdot\cdot\cdot$. Now, magnification of the peripheral powers or objects is readily and intuitively represented by adding a line or dot to each of the four extremities of the symbol, whereby it is converted into the simple swastika \oplus , or † .

Actually the figure is sometimes developed (as among the Pueblo peoples according to Cushing) by laying down four billets or arrows radiating from a fetishistic Middle toward the east, north, west, and south, and then adding, as the ritual proceeds, shorter transverse sticks touching the extremities of the four cardinal billets; the whole being done in such a manner as to harmonize ritual and symbol, and impress the former by the objective representation in the latter. In any case, the symbol is raised from its original value of $4 + 1$ to $8 + 1$; and the graphic representation accords with the shadowy concept lying behind the number-system in which the mystical Middle is persistent, and can be counted but once howsoever the value be augmented. Similarly the peripheral potencies may be multiplied by the addition of dots, as in a common form of the swastika noted by Wilson,  or ¹ or by the development of the "meander," , which thus represent $12 + 1$, $20 + 1$, and $20 + 1$; and the augmentation may proceed indefinitely, by either mechanical or mental addition, though always in accordance with the primary principle that the Middle is reckoned but once.

The law of augmentation in the senary-septenary system is similar. When the concept is directional, as in that form of the Cult of the Quarters in which zenith and nadir are reckoned as cardinal points, the mechanical symbol is complicated and eventually modified through the difficulty of depicting tri-dimensional relations on a bi-dimensional surface. Among the Pueblo peoples

¹ *The Swastika*, Rep. U. S. Nat. Museum for 1894, p. 767. Wilson, following Max Müller and Burnouf, notes that the additional billets or bars completing the swastika proper may be turned either to right or to left (i. e., the development of the figure may be either clockwise or counter-clockwise), but properly questions whether distinct names should be given the forms. In view of the fact that habitual motions of primitive peoples are predominantly centripetal or toward the body, while the predominant motions of advanced peoples are centrifugal, it seems probable that the clockwise swastika represents the higher cultural plane (e. g., of writing toward the right), and would therefore be normal if the symbol itself were normal to advanced culture; but since the symbol pertains in all essential respects to the culture-stage characterized by centripetal hand movement, the counter-clockwise form would seem to be more fairly considered the normal one.

this difficulty is overcome by bisecting two of the quadrants in a simple cruciform symbol in such manner as to produce the asymmetric figure ; but the ever-acting mechanical tendency operates to produce the regular figure  as the applications of the systems are extended. In either case, augmentation is effected by doubling or further increasing the peripheral extremities in such manner as to produce simple hexagrams, at first irregular, , and eventually regular, , or . The value of successive augmentations is expressed by the figures $6+1$, $12+1$, $18+1$, etc., i. e., by successive additions (mechanical or mental) to a once-reckoned Middle.

Now, comparison of these two number-systems, especially as illumined by the Pueblo method of depicting the fifth and sixth directions, indicates that the higher is produced from the lower simply by the superposition of a binary system on the quaternary system; and the inference, coupled with the patent fact that the higher base is the measure of increased intellectual capacity, seems to define the course of development of both systems. True, it is difficult for the arithmetical thinker to see how the mathematical pioneer missed the now-plain road from the indefinite quaternary-quinary notion to the definite quinary concept; but the fact cannot be gainsaid that the road *was* missed by many primitive tribes of especially mystical cast of mind, and that it was found and followed only by the ancestors of the practical Arabs with their decimal system, the barefoot Mexicans with their vigesimal system, and a few other peoples of exceptionally vigorous mind. The failure to find so plain a way may be ascribed largely to the complete domination of primitive thought by mystical concepts; and it would seem to repeat the demonstration by other facts that throughout much of prescriptorial culture little if any use was made of nature's abacus, the ever present human hand—for a habit of finger-counting could hardly fail to fix the quinary system in the minds of counters able to grasp so high a number as five without aid of extraneous symbols.

The development of the senary-septenary system out of the quaternary-quinary arrangement forcibly suggests the genesis of the latter; for it is but the sum or product of binary-ternary systems superposed by almacabalic augmentation. This suggested genesis would seem to be established by the occasional advances to the higher plane attested by some of the Australian numerations, as well as by the vestiges of the binary-ternary system along various culture lines, notably the Mongolian and Aryan. The presumptively primeval system apparently arose spontaneously, and became fixed through habitual mental effort shaped less by purpose-wrought symbols than by personal and subjective associations. Analogy with the higher systems (simplified to meet the dull mentation of the Blackfellows) would indicate that their number concept might be symbolized by any regular trigram uniting the perceived pair of objects to the unapperceived Ego; but the inequality of all social pairs in the tribal organization, the ever-varying relative potency of the good and evil mysteries, the unequal rank of the two ghostly Doppel-ichen, and other facts would suggest that a better figure for the concept would be an irregular trigram. But howsoever the system be represented graphically, the law of augmentation of the two higher systems prevails, as indicated both by certain of the Australian number-terms and by Mongolian vestiges, i. e., the augmentation proceeds by successive additions to a once-reckoned Middle, yielding the values, $2+1$, $4+1$, $6+1$, $8+1$.

So it seems feasible to define an archaic almacabala, including a method of using integral numbers rather as tokens of extra-natural potencies than as symbols for natural values and combining them by a simple rule tending to develop into algorismic processes, and including also a method of representing the numerical combinations by mechanical devices tending to develop into geometric forms; the system being characterized by the method of reckoning from an ill-defined unity counted but once in each combination.

IV

The course of intellectual development defined by the three prescriptorial number-systems (2-3, 4-5, 6-7) naturally leads interest toward the inception of the number idea—a point which must always remain obscure, save as illumined by analogies with lowest men and higher animals. Now, the more intelligent feral animals and the lowest known savages are fairly comparable in their capacity for counting; they are also alike in another respect of such consequence as to shape the character of both—their lives (as Ernest Seton Thompson so well shows for the animals) are lived in the shadow of tragedies unto often early and always unnatural death. This great fact of inevitable tragedy overlays all other facts woven in the web of nascent mind; the most firmly fixed habit of lowly life is that of eternal vigilance; the ever-present thought is that of ever-present danger; the dominant motive is that of mortal fear. No line of intellectual development can be fairly traced without full recognition of the ceaseless terrors of feral life; and the primeval interpretations of environment by animals and men alike manifestly reflect their tragic experiences: The fear-born cunning of the fox engenders that care for a way of escape without which he ventures on no advance; his every intuition is molded by living realization of a two-side universe—the Danger side in van, the Safety side in rear—with Self as the all-important center; and only religious adherence to experience-shaped instincts enables him to survive and permits his tribe to increase. The sagacious crow, even in semi-domestication, constantly betrays his notion of a two-side cosmos in frequent backward glances as he surveys the novel or forbidden field in front; and he is an arrant mystic, crazed with abject terror by night, and given to the formless fetishism of hoarding uncanny things in well-hidden shrines.¹ In like manner nearly all animals, from the fiercest carnivores to the timidest herbivores, manifest constant realization of three overshadowing

¹ *Wild Animals I have Known*, by Ernest Seton Thompson, 1898, pp. 72, 83.

factors in nature as they know it—factors expressed by Danger : Safety :: Self, or by Death and Life to Self, or in general terms, the Evil of the largely unknown and the Good of the fully known coördinated in the vaguely-defined Subject of the Badness and the Goodness; and the chief social activities of animal mates and parents are exercised in gathering their kind into the brightness of the known, and educating their native dread of all outer darkness. So, too, the more timid tribesmen of different continents betray, in conduct and speech, a dominant intuition of a terrible Unknown opposed through Self to a small but kindly Known. This intuition is not born of intertribal strife, since it is strongest in those innately amicable family groups who (despite the implication of their designation) typify lower savagery, and since it is slowly modified with the rise of self-confidence among vigorous and aggressive tribes in whose minds the Good grows large with the wax of conscious power; it is merely the subjective reflection of implacable environment—yet it is vaguely personified as a grisly and horrent bestial power, flaunting specters of death by tooth and claw, by serpent venom and swallowed poison, by pitiless famine and insidious disease, by wracking storm and whelming flood, by hydra-headed chance against half-felt helplessness; and over against this appalling Evil there is a less completely personified Good reflecting the small nucleus of confident knowledge with its far-reaching penumbra of faith. Accordingly, the lowest men and the higher animals seem much alike in their interpretation of nature—both rest their deepest convictions on a two-side cosmos connected in and through a largely-passive Self.

A vague yet persistent placement of the two ever-present Sides with respect to Self is clearly displayed in the conduct of animals and men—the Evil side is outward, the Good side at the place or domicile of the individual and especially of the group, as shown by the homing instinct of the wounded carnivore, by the haste of the fire-crazed horse to meet the flames in his

familiar stall, by human and equine nostalgia, and by the barbarian longing for burial in native soil. Moreover, both animals and men reveal indications of instinctive placement of the Sides in the individual organism; and the indications consistently point to persistent intuition of Face and Back as the essential factors of Self. Yet there is a significant diversity in the assignment of the Sides of the organism to the Sides of the Good-Bad cosmos: In general it appears that among the lower and the more timid the Back stands for or toward the Evil, the Face toward the Good, and that among the higher and more aggressive the Face is set toward the danger; e. g., defenseless birds and sheep huddle with heads together, savages sleep with heads toward the fire, and timid tribesmen tattoo talismans on their backs, while litters of young carnivores lie facing in two or more directions, self-confident campers sleep with feet to the fire, and higher soldiery think only of facing the foe. The interesting and significant growth of self-confidence need not be followed; it suffices to note that the primeval concept of the organic Ego, as revealed in the conduct of animals and men, appears to be that of a Face-Back (and not bilateral) unity, with the two Sides set toward the two aspects of a cosmos conceived in fear-born philosophy.

The passage of the primeval concept of a Face-Back Ego into that notion of two cardinal points suggesting a Cult of the Halves is happily represented among those Polynesian tribes who, according to Churchill,¹ have a system of geographic coördinates dominated by two cardinal directions, primarily seaward and landward, and secondarily northward and southward, respectively; while the language and customs connote a corresponding pantheon, capriciously malevolent on the sea-side and mildly benevolent on the

¹ Personal communication. While United States Consul at Samoa, Mr Churchill collected voluminous linguistic and other data well worthy of publication, though not yet issued. Conformably, Lesson and Martinet note that in Tahiti north and south are distinguished by denotive terms bearing a suggestive relation to tempestuous and milder winds, while east and west are without denotive designations, and are indicated only by descriptive phrases (*Les Polynésiens*, vol. II, 1881, p. 314).

land-side. This system of orientation is especially significant as a link in the chain of conceptual evolution, and equally as an explanation of the persistence of quasi-binary systems throughout Polynesia and Australasia with their shorelands of antithetic potencies; and no less significant are the facts in their bearing on the question of the habitat of primeval man, or of the orarian prototype already inferred from other facts.¹ Although varying from tribe to tribe in its relation to the meridian, this nascent orientation is no fleeting figment, but a deep-laid instinct so firmly rooted as to control every serious thought and direct every vital industry; indeed the Samoans and related navigators have developed their orientation into one of the most marvelous instincts in the whole range of animal and human life, viz: a cognition of definite albeit invisible sailing paths, whereby they are able to traverse the open Pacific, far beyond sight of land, with a degree of safety nearly equal to that afforded by chart and compass.

The Polynesian orientation at once illumines the unformulated Cult of the Halves, and opens the way to an explanation of the Cult of the Quarters; for each point of the shore is necessarily defined by sea in front and land in rear, and also by strands stretching toward the right and toward the left. Moreover, assemblages of Polynesians and Australasians, like the Iroquoian tribal councils, find it convenient to arrange themselves in coördinate groups or "Sides," so placed laterally as to face a speaker at the end of the plaza or prytaneum; and there is good reason for opining that the collective habit was soon strengthened, even if it was not initiated, by the slight asymmetry of the human body whereby the left brain receives blood a little more directly than the right and gives proportional excess of strength and cunning to the right hand. The initial inequality was doubtless too slight to yield more than barely perceptible physiologic advantage to the dextral fore-limb, as Brinton and Mason and others have shown; yet it may well have sufficed to set in operation a chain

¹ *The Trend of Human Progress*, Am. Anthropol. (N. S.), vol. I, 1899, p. 423.

of demotic interactions leading to the survival of the right-handed and the extinction of the left-handed throughout the earlier eons of human development. A clue to the demotic process is easily found in widespread horror of left-handedness, especially among primitive peoples; the clue becomes definite in the light of systematic infanticide among many tribes, whereby all manner of natal deformity is eliminated; it becomes conclusive in the light of the customs of those American tribes who habitually eliminate the sinistral offspring as monsters betokening the wrath of the powers. So, apparently initiated by slight physiologic difference and unquestionably intensified by demotic selection, right-handedness became even more predominant among primitive men than among their less superstitious descendants; the dexter and dextrous hand came to be exalted in scores of languages as "The One That Knows How" or "The Wise One," while the sinister hand was degraded by linguistic opprobrium unto a symbol of evil and outer darkness. Naturally and necessarily the bilaterally symmetric division of the Ego into Right and Left fell into superposition with the antecedent Face-Back concept, and produced a quatern notion such as that expressed in the Cult of the Quarters. Happily this transition is crystallized in the language of the Pitta-Pitta of Queensland, which possesses directional inflections indicating Front and Back reckoned from the Ego; and it is especially significant (in connection with the bimanual count inferred by W. E. Roth) that the inflection for Front applies also to (right?) Side.¹ And the quatern concept, born of unrecorded myriads of experiences, is much more than an idle fantasy of kiva and camp-fire. Intensified by the strongest motives of primitive life, it doubtless attained maximum strength before writing arose to divide its functions; yet, despite the decadence of millenniums, it is still expressed in two of the strongest instincts of higher humanity—the instinct of right-handedness, and the concomitant instinct of orientation.

¹ *Ethnological Studies*, op. cit., p. 2.

Accordingly, it seems safe provisionally to trace the origin of the number concepts in the light of common attributes of animals and men, and especially in the strong light afforded by the late-studied workings of primitive minds; and the lines seem clearly to define a crude philosophy whence all almacabalic and mathematical systems have necessarily sprung.

V

The character of almacabala, and the strength of its hold on the human mind, are illustrated by numberless vestiges, mainly mystical numbers and cognate graphic symbols. The entire series of mystical numbers may readily be ascertained by juxtaposing the three almacabalic number-systems and the products of their augmentation under the almacabalic rule. They are as follows (the super-mystical numbers accentuated):

2-3- 3, 5, 7, 9, etc.

4-5- 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, etc.

6-7- 7 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73, etc.

The vestigial uses of the binary-ternary system are innumerable. Two persists as the basis of the semi-mystical Aristotelian classification, which still exerts strong influence on Aryan thought; two is the basis, also, of the largely-mystical Chinese philosophy in which the complementary cosmologic elements, Yang and Yin, are developed into the Book of Changes¹; and it finds expression, either alone or in its normal union, in most Aryan cults. The mystical three pervades nine-tenths of modern literature and all modern folklore; it finds classic expression in the Graces and the Fates; it is particularly strong in Germanic and Celtic literature, cropping out in the conventional Three Wishes and Three Tests (a survival of the ordeal), and also as a customary charm number; and in these or related ways it persists in half the families and most of the child-groups even of this country and of today. The concept survives, also, in all manner

¹ *Chinese Philosophy*, by Paul Carus, 1898, p. 3 et seq.

of trigrams—triangles, triskelions, hearts, etc.—of mystic or symbolic character.

The quaternary-quinary system survives conspicuously in the form of graphic devices, especially the world-wide cruciform symbol, which has taken on meanings of constantly increasing nobility and refinement with the growth of intelligence. Hardly less conspicuous are the classic and later literary survivals in the Four Elements (Air, Earth, Fire, Water) of alchemistic philosophy, the Four Winds of astrology and medieval cartography, the Four Iddhis of Buddha, and the Four Beasts of Revelation and their reflections in the ecclesiastic writing of two millenniums; while the survivals in lighter lore are innumerable. The system persists significantly also in its augmentals, especially nine, thirteen, twenty-five, forty-nine, and sixty-one. The numerical vestiges are naturally for the most part quaternary, since the quinary aspect is merged and largely lost in algorism.

The senary-septenary system survives as the bridge connecting almacabala and mathematics. In the graphic form it became Pythagoras' hexagram of two superposed triangles, the equally mystical hexagram of Brianchon with which Paracelsus wrought his marvels, and the sub-rational hexagram of Pascal, while the current hexagram of the Chinese is apparently a composite of this and the binary as well as algorismic systems. In the numerical form, six and (more especially) seven play large roles in lore and in the classical and sacred literature revived during the Elizabethan period; even so recently as the middle of the century the hold of the astrologic seven was so strong as to retard general acceptance of the double discovery of the eighth planet, Neptune; and equally strong is the hold on the average mind of certain senary-septenary augmentals, particularly those coinciding with the augmentals of the lower systems.

In tracing vestiges in the form of augmentals, it is clearly to be borne in mind that their significance, like that of the primary numbers, is mystical rather than quantitative, so that certain

augmental numbers possess greater vitality than others of corresponding arithmetic grade. This is especially true of the *almacabalic* doubles, notably nine as the first augmental of five and thirteen as that of seven; for in these and other cases the first augmental is commonly of opposite sign (in *almacabalic* sense) from its basis—e. g., five and seven are beneficent or “lucky,” while nine and especially thirteen are maleficent or “unlucky” numbers. Moreover, there is a further mystical intensification in squares of the bases (perhaps growing out of mechanical or arithmetical superpositions on the mystical notions); and the charm seems to be still further augmented by coincidences between the several systems. It is partly through this mystical accentuation of the always mystical augmentals that such numbers as nine, thirteen, forty-nine, and sixty-one become conspicuous as factors and vestiges of *almacabala*.

Nine survives as a mystical number in the Muses of classical mythology, in Anglo-Saxon aphorisms emphasizing the vitality of the cat and the effeminacy of the tailor, and as a recurring tale in all of the superabundant Celtic lore such as that recently recorded by Seumas MacManus; it even survived in the school-books of the early part of the century in the more curious than useful arithmetic process of “casting out the nines”; and throughout the present decade the newspaper-writing jugglers with nines have found (and diffused) much mystery-tinged amusement in *almacabalic* analyses of the numbers 1890-1899.

Glaringly prominent in the mythology of the century is the bode clustering about the ill-omened first augmental of “lucky” seven; indeed it is probable that nearly half of the enlightened citizens of the world’s most intelligent country habitually carry the number thirteen in their minds as a messenger or harbinger of evil. The *almacabalic* double of thirteen (which is at the same time an augmental of five) has largely lost its mystical meaning in Europe and America, apparently through friction against practical arithmetic; but it retains no little hold on the oriental mind,

and finds expression in twenty-five-fold collectives in India and China, and in a rather frequent organization of Tibetan tribes into twenty-five septs or formal social units. Eminently conspicuous in Europe and America is the mystical number forty-nine, especially when expressed as seven \times seven; for, in the belief of a large element of European population, the seventh son of a seventh son needs no training to fit himself for medical craft, while scanners of advertising columns of American newspapers may daily read anew that the seventh daughter of a seventh daughter is a predestined seeress.

Few of the larger mystical numbers have survived the shock of occidental contact; but they abound in the Orient. The coincidental-augmental sixty-one prevails in Tibet, where Sven Hedin found a lama, one out of sixty-one of coördinate rank, who professed survival for sixty-one millenniums, through a succession of exoteric deaths and esoteric reincarnations at uniform periods of sixty-one years;¹ and this odd value is explained by the designation of the sixty-first figure in the Mongolian hexagram—"The Right Way" or "In the Middle"²—which at the same time connects the Book of Changes with the nearly world-wide Cult of the Quarters and its mystical Middle. The numbers sixty-three and sixty-five are also mystical in Chinese philosophy, though their potency would seem to be dwarfed by the mechanical-arithmetical structure of the octonal square to which they have been adjusted evidently during recent centuries. Among the Hindu more or less mystical numbers abound, and many of these are found on analysis to correspond with conventional albacabalic augmentals and coincidentals; while the Buddhistic rituals and series of aphorisms often run in measures of fives, with an initial or final supernumerary—the feature being apparently fixed by a mnemonic finger-count superposed on the almacabalic system, much as the octonal count is superposed on the mystical figures in the Chinese hexagram.

¹ *Through Asia*, by Sven Hedin, 1899, vol. II, p. 1132.

² *Chinese Philosophy*, op. cit., p. 12.

Suggestive vestiges of the mystical number-groups persist widely in the form of irrational and functionless supernumeraries, such as the thirteenth loaf in the baker's dozen, the twenty-first skerret in the coster's score, the thousand-and-first night of Arabian tale, and the conventional overplus in the legal "year and a day." It is possible that the supernumerary habit was crystallized in some cases by simple object-counting so conducted as to include an additional object as a tally; but there are many indications that the habit originally sprang from *almacabalic* augmentation, in which the sum is always one more than the multiple. Moreover, the supernumerary habit is especially characteristic of countries and culture-stages in which mystical number-jumbles are rife.

The various vestiges (which are far too many for full enumeration) at once illumine pre-rational numeration, and establish the course of development of number concepts suggested by the customs of peoples still living in the lower culture stages; while conversely the definition of *almacabala* serves to explain certain curious vestiges of primitive thought prevailing even today and in the highest culture.

VI

The way from alchemy to chemistry was long and devious, as shown by a voluminous literature worth scanning only as a means of tracing the growth of knowledge; the way from astrology to astronomy was still longer and more devious, as shown by loose straws of both literature and lore; and the way from *almacabala* through algorism to the rational science of quantities must have been longest and most devious of all. Yet it is worth while to gather and arrange the shreds of record and tradition which alone remain to mark the original way, and to compare them with the more abundant remnants of similar lines now followed by lower races—for these shreds, scanty though they be, define the birth of science.